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BRIEF COMMUNICATION

# MATHEMATICAL METHODS FOR THE EQUATIONS OF KINETICS OF GASES
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Approximately forty mathematicians and physicists gathered for this informal workshop in Paris. The workshop was organized by the Institute for Applied Mathematics of the École Normale Supérieure under the supervision of Professor C. Bardos.

The École Normale Supérieure, E.N.S., as many institutes in France are denoted by abbreviations, is situated in some buildings across the street from each other near the Panthéon and the Jardin du Luxembourg, with convenient subway connections to the airport Charles de Gaulle and the train stations Gare du Nord and Gare de Lyon. The institute itself lacks an elevator, but has convenient office areas, a conference room and a terrace with a terrific view of the neighbouring parts of Paris. It is surrounded by a great variety of restaurants and bars, so one need not get bored. The conference was held across the street in a lecture room equipped with an overhead projector and ample blackboard room.




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The attendance of the workshop was originally limited to the French Boltzmann community plus a few invited speakers from abroad, such as Profs. J.T. Beale from Duke University, R. Caflish from Courant Institute and B. Nicolaenko and E.W. Larsen from Los Alamos National Laboratory, all of them residing in the United States. Further, Profs. K. Asano from Japan, J. Batt and J. Voigt from the Federal Republic of Germany, C. Cercignani from Italy, and probably some others we forgot to mention. In spite of the limited effort to advertize this workshop, it soon attracted the attention of several other people in the field and as a result it expanded to its eventual size of 34 speakers.

The purpose of the workshop was to bring together mathematicians, physicists and engineers interested in the nonlinear and the linearized Boltzmann equation. In this respect the conference has been a great succes. Five talks involved discrete velocity models of the (nonlinear) Boltzmann equation (those by Profs. H. Cabannes, J.T. Beale, S. Kawashima, R. Gatignol and L. Tartar). Several other talks dealt with radiative transfer in stellar atmospheres (those by Profs. R. Sentis, F. Golse and B. Perthame), which topic was virtually absent from the conferences at Oberwolfach and Montecatini. Another series of talks dealt directly or indirectly with the issue of well-posedness of time-dependent and stationary linear transport equations (those by Profs. J. Voigt, M. Mokhtar-Kharroubi, P. Degond, W. Greenberg, C.V.M. van der Mee, B. Nicolaenko, H. Emamirad and P. Nelson) and still another set of talks with nonlinear existence theory, this time for nondiscrete models (those by Profs. K. Asano, J. Batt, H. Babovsky, T. Gustafsson, C. Cercignani, K. Hamdache, R. Caflish and H. Cornille). A series of lectures gave attention to a great variety of numerical aspects (those by S. Mas-Gallic, U. Frisch, A. Badruzzaman, J. Ullo, B. Mercier, E.W. Larsen and P. Lascaux). A number of talks were on the periphery of the main topic of the conference. G.H. Cottet applied vortex methods, Y. Pomeau described shock waves and P.F. Zweifel discussed a generalization of the Spectral Theorem. A few talks offered the participants a break from the mathematics prevailing in most contributions. U. Frisch gave an account of the nature and applicability of supercomputers, A. Badruzzaman presented well logging of boreholes as a hitherto virtually unknown application of transport theory, and J. Ullo discussed the mathematical model required for describing the liquid-glass transition. Summarizing, the workshop has presented a plethora of topics, most of them quite mathematical. For whomever was interested in well-posedness issues or in the more theoretical numerical questions, this workshop must have been most rewarding.

On the organizational side, one of the nice features of this conference was the observance of five to ten minute breaks between consecutive lectures, very helpful for maintaining attention and for intercepting lecturers right after their talks. The workshop was conducted in a most relaxed and stimulating atmosphere. We hope that this conference is going to be a biannual event, complementing the Blacksburg, Oberwolfach and Rarefied Gas Dynamics Conferences. In this way everyone interested would have a valid excuse to visit the food capital of this planet at least once every two years.

Monday June 3$^{rd}$, first thing in the morning, the workshop was opened by Prof. M. Dautray. On this day seven talks were given under his and Prof. C. Bardos' chairmanship.

H. Cabannes (Lab. Mech. Théor. Paris VI) gave a review of the derivation, the conservation laws, particular examples, the H-theorem and the existence theory for the solutions of the discrete velocity Boltzmann equations. Introducing the transition probabilities $A_{ij}^{k\ell}$ arising from a collision where the velocities indexed by i,j change into those indexed by $k,\ell$, and accounting for free streaming, gain and loss terms he derived the equation

$$\frac{\partial N_i}{\partial t} + \vec{u}_i \cdot \nabla N_i = \frac{1}{2} \sum_{jkl} A_{ij}^{k\ell}(N_k N_\ell - N_i N_j), \tag{1}$$

discussed some special cases and the symmetries of $A_{ij}^{k\ell}$. He went on to discuss the definition of the macroscopic quantities, the collision invariants, the conservation laws and their properties and the H-theorem. The latter was explained in connection with the existence of a (unique) Maxwellian state. Finally, he gave an account of the corresponding Euler equations, the semi-discrete Boltzmann equation (where the speeds but not the directions are discretized) and the few known global existence results.

J.T. Beale (Duke University) discussed the one-dimensional Broadwell model

$$v_t + v_x = z^2 - vw, \tag{2a}$$

$$w_t + w_x = z^2 + vw, \tag{2b}$$

$$z_t = \frac{1}{2}(vw - z^2), \tag{2c}$$

as well as more general discrete velocity models. It is known that integration along characteristics and the contraction mapping principle lead to the local existence, uniqueness and nonnegativity of the solution of Eqs. (2) for nonnegative initial data in $L^1 \cap L^\infty \cap C^1$. Using a global (in time) existence result of Crandall and Tartar, some asymptotic formulas for $t \to \infty$ were derived. For instance, there exist $v^\infty(x)$ and $w^\infty(x)$ in $L^\infty$ such that for $1 \le p < \infty$

$$\|v(\cdot,t)-v^\infty(\cdot-t)\|_p \to 0, \tag{3a}$$

$$\|w(\cdot,t)-w^\infty(\cdot+t)\|_p \to 0, \tag{3b}$$

$$\|z(\cdot,t)\|_p \to 0, \tag{3c}$$

while $(v^\infty, w^\infty)$ depends continuously on the initial data. A similar asymptotics result was derived in $L^\infty$. The results were extended to a class of one- and three-dimensional discrete velocity models satisfying mass and energy conservation, the H-theorem and a technical "square condition". The latter condition excludes the Carleman model (where Eqs. (3) do not hold true anyway) from consideration.

S. Kawashima (Nara University) departed from the discrete velocity model

$$\frac{\partial F_i}{\partial t} + \sum_i v^i \nabla_x F_i = Q_i(F,F),$$

where

$$Q_i(F,G) = \sum_{jk\ell} \{A_{k\,\ell}^{i\,j} F_k G_\ell - A_{i\,j}^{k\,\ell} F_i G_j\},$$

and studied its linearization about a Maxwellian M satisfying $Q_i(M,M)=0$ for all i, which leads to a nonpositive symmetric linearized Boltzmann operator $L_M$. On considerating the time eigenvalue problem for the linearized equation and defining a compensating function to be an odd and skew symmetric $C^\infty$ function on $S^{n-1}$ for which the symmetric part of the matrix $K(\omega) \sum_{j=1}^{n} V^j \omega_j + L_M \gg 0$, he proved that the existence of a compensating function is equivalent to the strict negativity of the real parts of the time eigenvalues. For the latter a "pointwise" strict negativity condition was shown equivalent to a "uniform" strict negativity condition. An example was given of a linear Boltzmann type equation, for which the linear manifold of collision invariants is eight- rather than five-dimensional.

K. Asano (Kyoto University) considered the zero Mach number (in his notations, $\lambda\to\infty$) limit of the compressible Euler equation, writing the pressures at times t and t=0 as

$$p(t,x) = \bar{p}\{1+\lambda^{-1}q(t,x)\},\ p_0(x) = \bar{p}\{1+\lambda^{-1}q_0(x)\},$$

and studied equations involving q and $q_0$. It was shown that for small initial data $u_0=(q_0,v_0)\in H^{\ell}$, $\ell\geq 3$, and $|u_0|\leq a$ ($\leq 0.2$) the solution of the compressible Euler equation exists uniquely, satisfies certain regularity and boundedness conditions and converges (as $\lambda\to\infty$) to the unique solution of the incompressible Euler equation. Herewith he improved considerably a result of Kleinerman and Majda. An asymptotic expansion (for large $\lambda$) was given for the solution of the compressible Euler equation.

J. Batt (Universität München) studied the Vlasov-Poisson system

$$\frac{\partial\varphi}{\partial t} + v\nabla_x\varphi - \nabla_x u(t,x)\nabla_v\varphi = 0,$$

$$\nabla^2 u(t,x) = 4\pi\int\varphi(t,x,v)dv,$$

under spherical symmetry. The existence of a global classical solution (a result due to Horst and Hunze) was presented. Using the method of characteristics, the corresponding stationary problem was solved. The $t\to\infty$ limits of the averages of the kinetic and the potential energy over [0,t] were shown to exist and to be finite. A short discussion was given of the relativistic case.

S. Mas-Gallic (C.E.A. Limeil) introduced an approximation of solutions of (integro)differential equations by replacing integrals by expressions of the form

$$\int_{\mathbb{R}^n}\psi(x)\mu(x)dx \simeq \sum_k \omega_k\psi(\xi_k)u(\xi_k),\ u \simeq \sum_k \omega_k u(\xi_k)\delta(x-\xi_k),$$

and the equations for the characteristics of the first order differential operators by

$$(\partial_t\cdot + \sum_{i=1}^n \partial_i(a_i\cdot))\alpha(t)\delta(x-x(t)) = \frac{d\alpha}{dt}(t)\delta(x-x(t)).$$

In this way she was able to approximate diffusion and integral terms in kinetic equations by replacing the solution by a discrete sum containing $\delta$-type distributions.

She proved that certain conservation laws are left unaltered by the approximation and that the approximative solutions converge to a solution of the original equation. This method was then shown to be applicable to the compressible Euler equation and the incompressible Navier-Stokes equation, as well as to the linearized Boltzmann equation.

The second day took place under the chairmanship of Profs. J.T. Beale and P.F. Zweifel.

J. Voigt (Universität München) considered the neutron transport equation

$$\frac{\partial f}{\partial t} = -(\xi \cdot \nabla_x f + hf) + \int k(x,\xi,\xi')f(t,x,\xi')d\xi' = Tf + Kf \tag{4}$$

for $x \in D \subset \mathbb{R}^n$, $\xi \in V \subset \mathbb{R}^n$, $h \geq 0$, $k \geq 0$ (but possibly unbounded $k$) and vacuum boundary conditions, in the Banach lattice $L_1(D \times V)$ for a nonmultiplying medium. This equation was shown to be well-posed with its solution given by means of a positive and contractive (i.e., substochastic) $C_0$-semigroup. An instructive example was given where, in spite of singularities in the integral kernel, the transport operator on the domain of functions differentiable (in the distributional sense) along characteristics and satisfying the boundary conditions, and not some nontrivial closed extension, generates the semigroup describing the solution. One should notice that in this case the transport operator is not a bounded additive perturbation of a known semigroup generator.

H. Babovsky (Universität Kaiserslautern) proved the global existence of the solution of the nonlinear Boltzmann equation for a hard sphere gas in a bounded region for small initial data bounded above by a Maxwellian, using the upper-lower solution method of Kaniel and Shinbrot. Next, he described the Knudsen flow through a cylinder by a stochastic process and showed that for cylinder double radius $d \to 0$ and $d\sqrt{T}$ constant, where $T$ is the temperature, the flow can be described by the diffusion approximation. Finally, he modelled a test particle in a dense half-space gas with a partially absorbing boundary by a stochastic process. He went on to prove that, if the probability of recurrence of the particle to the wall is strictly less than one, the relative loss of particles up to a fixed time $\Delta t$ converges to one as the mean free path vanishes.

U. Frisch (Observatoire de Nice), describing joint work with B. Hasslacher (Los Alamos) and Y. Pomeau (Saclay), began his disposition of discrete kinetic equations and supercomputers by emphasizing the need of parallel processing to overcome the intrinsic limitation of computers that signals cannot travel any faster than the speed of light. Parallel processing was illustrated on discrete velocity models. Two of these models were discussed in particular. In the first model one has a square velocity lattice with molecules moving with uniform speed and discrete time, where in one time unit each molecule either moves on by one position in its original direction or meets a molecule from the opposite direction, after which they move on perpendicularly. To overcome the lack of Galilei invariance and the detrimental effect of higher order terms in the corresponding nonlinear Boltzmann equation, a second model was introduced, where the velocities form a triangular equilateral lattice and on binary head-on collisions the particles leave with equal probability in opposite directions over $60^\circ$ or $120^\circ$, while on ternary collisions every particle is reflected back. Finally, the relative merits of the supercomputers XMP $64^3$ and Cray2 $256^3$, each of them having parallel processing sites in each perpendicular direction, were discussed.

M. Mokhtar-Kharroubi (Université de Besançon) gave a description of the behaviour of the time eigenvalues of the time-dependent linear transport equation. On denoting the free streaming plus cross-section operator by T (adopting vacuum boundary conditions) and the collision operator by K (assumed position-independent and positive self-adjoint in $L_2$-velocity space, i.e., $k(x,\xi,\xi')=k(\xi,\xi')=k(\xi',\xi)$ and $\int k(\xi,\xi')f(\xi)f(\xi')d\xi d\xi' \geq 0$), $B_\lambda=(\lambda-T)^{-1}K$ power compact for $\mathrm{Re}\lambda>\lambda^*=-\inf\sigma(\cdot,\cdot)$, a necessary and sufficient condition was given for the existence of a dominating eigenvalue $\lambda_0$ with $\lambda_0>\lambda^*$. This condition was shown to be fulfilled for sufficiently large spatial domains. Estimates for the minimal number of real eigenvalues $\lambda>\lambda^*$ were given. More refined results were presented for isotropic scattering and constant cross-section.

A. Badruzzaman (Schlumberger, Inc., Richfield, Connecticut) gave a review of well logging techniques to determine the constituent material of a borehole, using a neutron source in combination with a near and a distant detector. The mathematical model amounts to an (inverse) transport calculation, which must be performed within a 1% accuracy. The comparative merits of the most generally used Monte Carlo method and the newly proposed discrete ordinates method were discussed, and some first numerical results obtained by discrete ordinates were given. Special emphasis was given to the discussion of the immense computer storage needed for a discrete ordinates calculation having the required accuracy. A new three-dimensional discrete ordinates algorithm was described briefly.

P. Degond (Polytechnique de Paris), reporting on joint work with S. Mas-Gallic (C.E.A. Limeil), proved the unique solvability of the stationary Fokker-Planck problem

$$\mu\frac{\partial f}{\partial x} - \sigma\frac{\partial}{\partial\mu}((1-\mu^2)\frac{\partial f}{\partial\mu}) = R(x,\mu),\ x\epsilon[0,L],\ \mu\epsilon[-1,1], \tag{5a}$$

$$f(0,\mu) = \varphi_+(\mu),\ \mu>0, \tag{5b}$$

$$f(L,\mu) = \varphi_-(\mu),\ \mu<0, \tag{5c}$$

in a weighted $L_2$-space, using the unique solution of the time-dependent problem. He also proved the convergence of the solution of the $\epsilon$-scaled version of Eqs. (5) to the corresponding diffusion approximative solution as $\epsilon\to0$. Finally, two methods were discussed to compute the extrapolation length $\Lambda$ for the Milne problem, where $L=\infty$, $\varphi_+(\mu)\sim\mu$ and $f(L,\mu)=O(L)$ as $L\to\infty$. One method mimicked the invariant imbedding procedure of Chandrasekhar and reduced the problem to a partial differential equation with boundary conditions. The second method used the expansion of the solution in the eigenfunctions of the full-range stationary equation (i.e., Eq. (5a) for $x\epsilon\mathbb{R}$). The value of $\Lambda$, $\Lambda\approx0.718$, appeared to be surprisingly close but not equal to the corresponding value for isotropic neutron transport.

On wednesday it was the turn of Profs. P. Nelson and H. Cabannes to precede the workshop.

W. Greenberg (Virginia Tech) gave a review of some recent results on solutions of the stationary transport problems

$$T\psi'(x) = -A\psi(x),\ 0<x<\infty, \tag{6a}$$

$$Q_+\psi(0) = \varphi_+, \tag{6b}$$

$$\|\psi(x)\| = O(1)\ \text{or}\ o(1)\ (x\to\infty), \tag{6c}$$

in spatially homogeneous half-space media. Althouth the Beals-Hangelbroek semigroup approach has provided a rather complete understanding of the abstract boundary value problem with symmetric collision operator A in a Hilbert space setting, the theory for nonsymmetric A is still poorly developed. Greenberg outlined some recent joint work with A. Ganchev (Sofia, Bulgaria), which provides a more detailed understanding of the somewhat ad hoc methods in the Larsen resolvent integration scheme. The results are applicable to nonsymmetric collision operators and, more generally, in a Banach space setting.

J. Ullo (Schlumberger, Inc., Richfield, Connecticut) devoted a molecular dynamics study to dynamical fluctuations of the liquid-glass transition, which is generally thought to be a natural application of mode coupling theory. In the two coupled equations written down the quantity to be computed is the (time-dependent) structure factor $F(q,t)$. Two tests of the model were presented. The first test consisted of qualitative comparison with experiments, the second test of computer simulations, both on the fluid and on the glass side of the transition. The problems stemming from long computer run times, crystallization near the transition and the high frequency resolution needed to see double peaks were discussed.

C.V.M. van der Mee (Texas Tech; Clarkson University) gave a review of integral formulations of stationary transport equations in plane parallel finite or semi-infinite media, using an abstract framework as in Eqs. (6) and accounting for reflection by boundaries. For half-space media a general method for obtaining the solutions of the stationary problem was discussed, which makes use of generalized Chandrasekhar H-functions. A method was presented for constructing the projections and semigroups appearing in the (formal) stationary solutions, in the case of nonselfadjoint models. The existence of these H-functions was proved using the cascade decomposition method of linear systems theory.

T. Gustafsson (Chalmers University, Göteborg) derived $L^p$-estimates for the global solution of the spatially homogeneous nonlinear Boltzmann equation. The method consisted of an application of nonlinear interpolation to the $L^1$ and $L^\infty$ existence results of Arkeryd, Elmroth and Di Blasio.

C. Cercignani (Politecnico di Milano) established a global existence result for the nonlinear Enskog equation. The proof was based in part on the standard method used by Arkeryd on the space homogeneous nonlinear Boltzmann equation.

R. Gatignol (Lab. Mech. Théor. Paris VI) presented results on models of the nonlinear Boltzmann equation having a finite discretized set of velocities, but with continuous space and time dependence. The only restriction on the nonlinear interaction is that they are bilinear, that mass momentum and energy are conserved and that the H-theorem holds. For such models, she discussed boundary conditions, the fluid dynamics limit and shock waves, including the correct Rankine-Hugoniot and entropy conditions.

Y. Pomeau (C.E.A. Saclay; École Normale Supérieure, Paris) proposed a similarity form in $\xi$ and x, as the dominant part of the Boltzmann solution at the cold side of a strong shock. The similarity solution is believed to be correct for intermolecular forces with infinite collisional cross-section and for infinite Mach number, so that the temperature is zero at the cold end of the shock.

Thursday was devoted to radiative transfer, under the direction of Profs. R. Caflish and B. Mercier.

B. Mercier (C.E.A. Limeil) considered the equation

$$u - Pu = f,$$

where P is an integral operator, replacing u by $u_h \sim \sum_i \alpha_i \delta_{x_i}$, where $\{x_i\}_i$ are random points in a Monte Carlo calculation or numerical integration points in a deterministic particle method. In both cases weak convergence results may be established by considering a similar approximation of the solution of the adjoint equation

$$u^* - P^* u^* = f^*.$$

Some numerical results were given for a spherically symmetric stationary transport problem, which showed good agreement with the predictions of the above theory.

E.W. Larsen (Los Alamos) presented a comparative assesment of three finite difference schemes, which were illustrated by their relative success on the (exactly solvable) problem

$$\mu \frac{\partial \psi}{\partial x} + \sigma_r \psi(x,\mu) = 0, \ x \epsilon (0,\infty), \ \mu \epsilon [-1,1],$$

$$\psi(0,\mu) = 1, \ \mu > 0,$$

$$\lim_{x \to \infty} \psi(x,\mu) = 0.$$

The discretized solution (with x-mesh h) is only accurate if $\sigma_r \gg 1$. However, many problems with large cross-section require $\sigma_r h = O(10^4)$. For this reason the above problem was rescaled with scaling constant $\epsilon \ll 1$ and the solution expanded as

$$\psi = \psi^{(0)} + \epsilon \psi^{(1)} + \epsilon^2 \psi^{(2)} + \cdots.$$

To this rescaled equation he applied the diamond difference method, the Lund method and the step (upwind) differencing scheme, and subjected each method to an asymptotic analysis in the thick and in the thin limit, both for the cell average fluxes and for the cell edge fluxes. It appeared that no method gave satisfactory results for the cell edge fluxes in the thick limit, that diamond differencing worked well in the three other cases, that the Lund method only works well on the cell average fluxes in the thick limit, and that the step differencing scheme gives poor results in all four cases.

R. Sentis (C.E.A. Limeil) discussed the behaviour of the solution of the radiative transfer problem

$$\frac{\partial I}{\partial t} + \mu\frac{\partial I}{\partial x} + \sigma I - \sigma b(\varphi) = 0, \tag{7a}$$

$$\frac{\partial \varepsilon}{\partial t}(\varphi) + \langle\!\langle \sigma b(\varphi) - \sigma I \rangle\!\rangle = 0, \tag{7b}$$

plus initial-boundary conditions, where

$$\langle\!\langle \cdot \rangle\!\rangle = \frac{1}{2}\int_0^\infty \int_{-1}^{1} \cdot \, d\mu d\nu, \quad b(\varphi) = c\nu^3[-1+\exp(-\nu/\varphi^{1/4})]^{-1},$$

and usually $1 \leq \sigma \leq 10^6$, at the interface of two media with different opacities $\sigma$, assuming the incoming flux known. He proved Eqs. (7) to be uniquely solvable. The same thing was proved for the corresponding linearized equation (about the Planckian $b(\alpha,\cdot)$) and an explicit expression for the outgoing flux in terms of the incident flux and Chandrasekhar's H-function was obtained.

F. Golse (École Normale Supérieure, Paris), reporting on joint work with B. Perthame (E.N.S., Paris), derived a global existence result for the solution of the radiative transfer problem

$$\frac{\partial I}{\partial t} + \Omega \cdot \nabla_x I + \sigma(\nu,\varepsilon)\{I - B(\nu,\varepsilon)\} = 0, \tag{8a}$$

$$\frac{\partial \varepsilon}{\partial t} + \int_0^\infty \sigma(\nu,\varepsilon)\{B(\nu,\varepsilon) - I\} dv = 0, \tag{8b}$$

plus initial-boundary conditions, where for all $\nu > 0$ the function $B(\nu,\cdot)$ is increasing and positive. First an energy estimate and a minimax principle for the solutions were obtained. Next, he introduced certain technical assumptions on the opacity $\sigma(\nu,\cdot)$, rewrote Eqs. (8) as a vector-valued initial-value problem governed by an operator R and proved R to be T-accretive (in the sense of Crandall and Liggett). Next, he proved the existence and uniqueness of stationary solutions. Finally, he discussed possible extensions, such as Kramer's opacity $\sigma(\nu,\epsilon)=\{1-e^{-\nu/\epsilon}\}/\nu^3\sqrt{\epsilon}$, where one of the technical assumptions is violated.

B. Perthame (École Normale Supérieure, Paris), discussing joint work with C. Bardos and F. Golse (both E.N.S., Paris), applied rescaling to the boundary value problem

$$u + \Omega\cdot\nabla_x u + \sigma(\hat{u})(u-\hat{u}) = f, \tag{9a}$$

$$u|_{V_-} = h(x,\Omega), \tag{9b}$$

where $\hat{u}=|\delta|^{-1}\int_\Omega u d\Omega'$. On assuming $\sigma_\nu(T)$ decreasing in the temperature T, $\sigma(0)=+\infty$ and $\sigma(T)T$ increasing in T, he proved that the solution $u_\varepsilon$ of the scaled problem (9) converges in the $L_1$-norm to the solution $u_0$ of the problem

$$u_0 - \nabla^2 F(u_0) = \int_\Omega g d\Omega',$$
$$u_0|_{\partial V} \text{ given},$$

with suitable boundary data. Next, he reduced the nonlinear Milne problem connected with Eqs. (9) to a linear Milne problem and proved their unique solvability.

B. Nicolaenko (Los Alamos), reporting on joint work with C. Bardos (E.N.S., Paris) and R. Caflish (Courant), discussed the existence and uniqueness theory and the thermal layer solutions of the one-dimensional linearized Boltzmann equation

$$\xi_1\frac{\partial f}{\partial x} + Lf = 0,\ x>0,\ \xi\in\mathbb{R}^3,$$

for a hard sphere gas. Writing $L=\nu(\xi)+K$ where

$$\nu_0(1+|\xi|)\le\nu(\xi)\le\nu_1(1+|\xi|)$$

and K is compact, energy estimates were applied to prove the unique solvability of the half-space problem, assuming $\int\xi_1 M^{1/2} f d\xi=m_f$ given. In a similar way the unique solvability of the Milne problem was established. Finally, Benard's problem, i.e., a rarefied gas in a constant gravitational field, was discussed in short.

The last day comprised two sessions led by Profs. W. Greenberg and B. Nicolaenko.

H. Emamirad (Lab. Mech. Théor. Paris VI) applied the Lax-Phillips scattering theory to the linear time-dependent transport equation

$$\frac{\partial u}{\partial t} = Tu,\ T = T_0+A,\ A = A_1+A_2$$

in $L_1(\mathbb{R}\times V)$. A historical account was given of the scattering theory of linear transport equations and the spectrum of the free streaming operator $T_0$ (where he presented results in $L_2$ and $L_\infty$, supplementing Hejtmanek's $L_1$ results). After describing briefly the Lax-Phillips theory, he went on proving its conditions for a finite collision system.

P.F. Zweifel (Virginia Tech), reporting on joint work with C. Burnap (University of North Carolina, Charlotte), introduced a generalization of the notion of a scalar-type spectral operator, as studied by Dunford and Schwartz. In this generalization the spectral projections were unbounded operators on a common dense subspace.

P. Nelson (Texas Tech University), reporting on joint work with D. Seth (Los Alamos; Texas Tech), presented the stationary Spencer-Lewis equation

$$\mu\frac{\partial\psi}{\partial z}(z,\mu,E) - \frac{\partial(\beta\psi)}{\partial E} + \sigma\psi = \int_{-1}^{1} k(z,t,\mu,\hat{\mu})\psi(z,E,\hat{\mu})d\hat{\mu} + q(z,E,\mu),$$

which describes the continuous slowing down of electrons from a few MeV to a few KeV. A method for obtaining numerical solutions using an upstream energy differencing scheme was shown to lead to unique solutions having a limit as the differencing mesh decreases to zero, and to be numerically stable. Although the approximating difference equation was proved to be well-posed for a sufficiently fine energy grid, the well-posedness of the original Spencer-Lewis equation remained an open problem.

K. Hamdache (E.N.S.T.A. Palaiseau) considered the initial-boundary value problem

$$\partial_t f_i^\varepsilon + v\cdot\partial_x f_i^\varepsilon - \varepsilon L_i f_i^\varepsilon = Q(f_i^\varepsilon),$$

$$f_i^\varepsilon\big|_{t=0} = \varphi_i,$$

$$(Qf)(t,x,v) = \int_{S^2\times\mathbb{R}^3} B(\theta, |v-w|)\{f(v')f(w')-f(v)f(w)\}d\theta\, d\bar{v}\, d\bar{w},$$

where angular cut-off was applied and alternatively $L_1 = \nabla_x^2$ (artificial viscocity) and $L_2$ describes Fokker-Planck diffusion. For the initial value bound $|\varphi(x,v)| \leq m \exp[-(\alpha|v|^2+\beta|x|^2)]$ and m small, global existence of solutions for $\varepsilon=0$ and the convergence of the solution (as $t\to\pm\infty$) to a solution $g^{\pm}$ of the equation

$$(\partial_t + v\cdot\partial_x)g^{\pm} = 0$$

are known. Analogous results were obtained for $i=1,2$ and $\varepsilon>0$, also for small initial data only. For both cases it was shown that $f_i^{\varepsilon}\to f_i$ weakly and $Q(f_i^{\varepsilon})\to Q_i^*$, but not that $Q_i^* = Q_i(f_i)$.

L. Tartar (C.E.A. Saclay) gave a historical review of the existence and uniqueness theory of the one-dimensional Carleman and Broadwell models and other one-dimensional discrete velocity models. For such models global existence results, with small initial data in $L^1$, are known and these have been derived also for small data in $L^{\infty}$. Various techniques of obtaining such global solutions were discussed in detail.

R. Caflish (Courant Institute) discussed the solution of the nonlinear Boltzmann equation for two problems: a vapour layer between two liquids at different temperatures and the cold side of a strong shock wave. At small Knudsen number $\varepsilon$, the vapour layer solution is found by matching the Chapman-Enskog expansion (valid away from the boundaries) with boundary layers near the liquid-vapour boundaries. The boundary layer problem is solved as described in the talk of Nicolaenko. For a shock with infinite Mach number, the Maxwellian state at the cold side is a $\delta$-function, i.e., all the particles have the same velocity. The solution near the cold end is sought in the form

$$F(x,\xi) = \{1-a(x)\}\delta(\xi) + f(x,\xi)$$

with f being a regular function. Existence of solutions to weakly nonlinear ($a\ll 1$, $f\ll 1$) boundary value problems is proved. Although this does not solve the full shock problem, it nevertheless gives information about its positivity and its decay rate.

H. Cornille (C.E.A. Saclay) described special exact solutions of several Kač models for the spatially homogeneous nonlinear Boltzmann equation. The solutions consisted of finite sums of special functions. As they evolve in time, the solutions oscillate around the Maxwellian to which they eventually converge.

It remains to remark that the proceedings of this conference can be found in a future issue of Transport Theory and Statistical Physics.

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