Parameterized Scattering Matrices for Nonspherical Particles in Planetary Atmospheres

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Abstract

Parameterized scattering matrices that may be used for interpreting photopolarimetry of planetary atmospheres containing randomly oriented nonspherical particles are discussed. A particular parameterization that is especially useful for scattering by small particles, particles in the Rayleigh-Gans domain, and for scattering by a variety of irregularly shaped particles in the visible part of the spectrum is introduced. The use of parameterized scattering matrices is illustrated by deriving their elements as functions of the scattering angle from simulated measurements of the brightness and polarization of light reflected by a plane-parallel atmosphere containing spheroidal particles.

1 Introduction

Comparing results of measurements of the brightness and polarization of light scattered by an atmosphere with results of multiple-scattering calculations is very useful for obtaining information about the particles in that atmosphere. Often, such measurements are interpreted in terms of parameters describing the size, shape, and composition of the particles.

If no reliable a priori knowledge about the size, shape, or composition of the particles is available, or if the calculations of scattering matrices as functions of the scattering angle are prohibitively time-consuming in a trial-and-error-formulation, one might use parameterized matrices as scattering matrices. This may be considered as an extension of the well-known parameterization of the phase function by a Henyey-Greenstein function. Using parameterized matrices, measured quantities are interpreted in terms of the single-scattering behavior of the particles rather than in terms of their size, shape, and composition. Often, one can infer important physical constraints from derived parameterized matrices without knowing the precise shapes of the particles. Also, one may use single-scattering algorithms or measurements to interpret derived parameterized matrices in terms of particle properties.

Employing terms like *generalized single-scattering properties* [1], *synthetic phase matrix* [2], and *synthetic scattering matrix* [3], investigators have previously adopted similar approaches for interpreting polarization measurements of Saturn and Titan. In the present paper, we use constraints for parameterized matrices that were not taken into account by these investigators, and we give an illustration of the use of parameterized matrices in the interpretation of measurements.

2 Parameterization for common types of particles

Mishchenko et al. [4] give a comprehensive overview of measurements and calculations of light scattering by nonspherical particles covering a great variety of sizes, shapes, and compositions. This book chapter provides many useful references to papers dealing with scattering by mineral particles. On the basis of these papers we now define a class of parameterized matrices $\mathbf{F}(\Theta)$. In order to restrict the discussion to collections of randomly oriented particles which occur in equal numbers as their mirror particles, we first assume $F_{13}(\Theta) = F_{14}(\Theta) = F_{23}(\Theta) = F_{24}(\Theta) = F_{31}(\Theta) = F_{32}(\Theta) = F_{41}(\Theta) = F_{42}(\Theta) = 0$, $F_{21}(\Theta) = F_{12}(\Theta)$, and $F_{34}(\Theta) = -F_{43}(\Theta)$. Then, we parameterize

$$\begin{aligned}
F_{11}(\Theta) &= P_{\mathrm{HG}}(g,\Theta) = \frac{1-g^2}{(1+g^2-2g\cos\Theta)^{3/2}} , && (1)\\
\frac{F_{22}(\Theta)}{F_{11}(\Theta)} &= h + (1-h)\cos\Theta , && (2)\\
\frac{F_{33}(\Theta)}{F_{22}(\Theta)} &= \frac{2\cos\Theta}{1+\cos^2\Theta} , && (3)\\
-\frac{F_{21}(\Theta)}{F_{11}(\Theta)} &= p\frac{\sin^2\Theta}{1+\cos^2\Theta} , && (4)\\
\frac{F_{34}(\Theta)}{F_{11}(\Theta)} &= 0 , && (5)\\
F_{44}(\Theta) & \text{ is not specified,} && (6)
\end{aligned}$$

where Θ denotes the scattering angle and $P_{\mathrm{HG}}(g,\Theta)$ is the Henyey-Greenstein function. Here, g is the asymmetry parameter, p is the maximum degree of linear polarization for incident unpolarized light, and h is the relative contribution of the copolarized component to the total backscattered light if the incident light is completely linearly polarized.

Equations (1)-(6) meet all applicable requirements at $\Theta = 0°$ and $\Theta = 180°$ [5] if $-1 < g < 1$ and $0.5 \leq h \leq 1$. Further, a function $F_{44}(\Theta)$ exists so that the so-called "Cloude test" [6] is passed for all scattering angles when $|p| \leq 1$ if $h = 1$, and when $|p|$ is slightly less than $(1+h)/2$ if $h < 1$. Parameterized matrices that obey all these requirements will be referred to as *parameterized scattering matrices* (PSM's).

If the matrix elements $F_{11}(\Theta)$, $F_{22}(\Theta)$, $F_{33}(\Theta)$, $F_{21}(\Theta)$, and $F_{12}(\Theta)$ are expanded in generalized spherical functions of the scattering angle, the expansion coefficients decrease exponentially. The parameterization given in Eqs. (1)-(6) is especially useful for scattering by small particles, particles in the Rayleigh-Gans domain, and a variety of irregularly shaped particles in the visible part of the spectrum.

3 Parameterized scattering matrices in atmospheric analysis

3.1 Model and method

This section aims to illustrate the applicability of PSM's for interpreting photopolarimetry of a planetary atmosphere. In a case study, measurements of the brightness and polarization of reflected sunlight were simulated for a model atmosphere containing spheroidal particles. Subsequently, a PSM was derived that reproduced these measurements as closely as possible. This PSM was then compared with the original scattering matrix. This process is divided into two steps, as follows.

Step 1. We considered a homogeneous, plane-parallel atmosphere with an optical thickness of 1, bounded below by a completely absorbing surface, and containing randomly oriented oblate

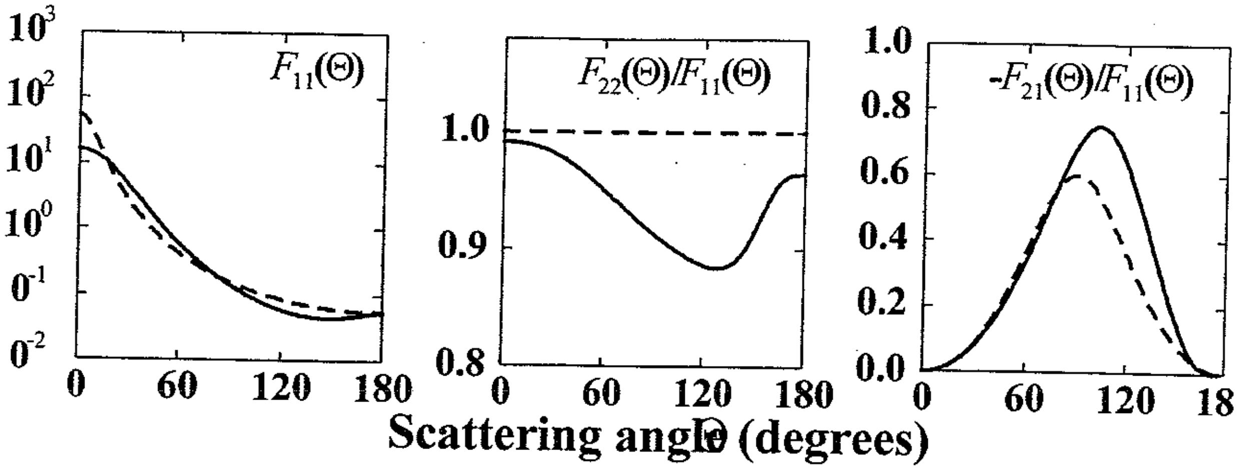

Figure 1: Solid: three elements of the original scattering matrix as functions of scattering angle. This matrix was calculated for a collection of spheroidal particles as described in the text. Dashed: the corresponding elements of the PSM that was retrieved from simulated measurements based on the original matrix.

spheroidal particles. These particles had a refractive index, m, of $1.33 + 0i$ and an aspect ratio of 3. The size distribution of the equal-volume spheres was chosen to be a gamma-distribution corresponding to an effective size parameter of 3 and an effective variance of 0.1. The scattering matrix of a volume element of these particles was calculated with the T-matrix method [7]. Three elements of the original matrix are shown as solid lines in Fig. 1.

Assuming a parallel beam of unpolarized light incident at the top of the atmosphere, we employed the adding/doubling method for calculating all four Stokes parameters of the light emerging from the top of the atmosphere for a number of viewing geometries. Geometries are characterized by $\mu_0 = -\cos\theta_0$, $\mu = \cos\theta$, and $\phi - \phi_0$, where θ_0 and θ are the zenith angles of the incident and scattered light, respectively, and ϕ and ϕ_0 are the corresponding azimuthal angles, measured clockwise when looking up. Values of 0.2 through 1.0 were considered for both μ and μ_0, with steps of 0.1; for $\phi - \phi_0$, values of 0° through 180° with steps of 30° were considered.

Step 2. Subsequently, we fitted results of calculations using the parameterization given by Eqs. (1)-(6) to these simulated measurements. Here, the same model was used as in Step 1, except for the scattering matrix of the particles. The albedo for single scattering equaled 1.

First, fixing p and h each at an initial value, the remaining parameter g was varied so that it minimized the function

$$\Delta I(g) = \sum_i [I_{c,i}(g)/I_{m,i} - 1]^2 \,, \tag{7}$$

where i numbers the geometries, and $I_{c,i}$ and $I_{m,i}$ are the calculated and "measured" brightnesses for geometry i, respectively. Then, conversely, g was fixed at its new value, and p and h were varied to minimize the function

$$\Delta P(p,h) = \sum_i [P_{c,i}(p,h) - P_{m,i}]^2 \,, \tag{8}$$

where $P = \sqrt{Q^2 + U^2}/I$. This process was repeated until the values of both ΔI and ΔP did not change by more than 0.0001 times their previous values.

3.2 Results and discussion

All elements of the retrieved PSM agreed fairly well with the original matrix elements. Three elements of the retrieved PSM are shown as dashed lines in Fig. 1. Although the retrieved function $F_{11}(\Theta)$ exhibits a more pronounced forward peak, the retrieved and original asymmetry parameters differ by only 3%. The retrieved and original functions $-F_{21}(\Theta)/F_{11}(\Theta)$ both show a large Rayleigh-like bell shape, but the retrieved maximum is 0.60 as opposed to the original maximum of 0.75. Note further that the retrieved PSM has $F_{22}(\Theta) = F_{11}(\Theta)$, in contrast with the original matrix, though the maximum difference is less than about 12%.

If we employ Mie theory with $m = 1.33 + 0i$ for interpreting the retrieved parameters, the retrieved asymmetry parameter is compatible with ensembles of spheres with effective size parameters of about 3. However, such spheres produce a much lower maximum of $-F_{21}(\Theta)/F_{11}(\Theta)$ than is retrieved here, which shows that the particles in our model atmosphere are nonspherical.

We continue our research, using other particles and parameterizations, to find out how well scattering matrices can be retrieved using PSM's. In addition, we plan to use PSM's for interpreting polarization data of Jupiter obtained by the Galileo spacecraft.

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