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A retrospective view on: “Testing scattering matrices: A compendium of recipes”

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ABSTRACT

The elements of matrices relevant to polarized light transfer often obey certain equalities and inequalities that can be used for testing purposes. As an introduction to the reprint of our paper (Hovenier and van der Mee, 1996 [7]) we first present the case history and significance of this paper and then discuss some later developments.

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Studies of scattering of electromagnetic radiation by particles are important in widely different parts of science and have an enormous number of practical applications. Taking polarization into account is often necessary, but not simple. Even single scattering of quasi-monochromatic radiation by one particle involves a real matrix with 16 elements. This so-called scattering matrix transforms the four Stokes parameters of an incident beam into the four Stokes parameters of a beam scattered in a particular direction. In his famous book on light scattering by small particles, first published in 1957, Van de Hulst [1] stated that there must be nine relations between the 16 elements, but he did not present these relations. In 1985 we decided to seek mathematical expressions and derivations for these relations. It turned out that, though some relations had been found earlier, a comprehensive treatment based on first principles was still missing. Such a treatment was published in 1986 [2], together with a plethora of equations for the elements of the scattering matrix, valid for a particle of arbitrary size, shape, structure and composition. It was also shown in that

paper that, if the light incident on a single particle is fully polarized, so is the scattered light.

In practice we often have to deal with an assembly of many independently scattering particles with different sizes, shapes, structures, compositions and orientations. Some inequalities for the scattering matrix of such an assembly had already been found before 1986, but a much larger collection was presented in [2]. In addition, it was proven that no element of the scattering matrix of an assembly of particles can have an absolute value larger than the element in the upper left corner, which is also known as the scattering or phase function.

In later investigations we found many relationships (equalities and inequalities) for other matrices relevant to polarized radiation, such as the phase matrix of an assembly of randomly oriented mirror symmetric particles [3] and general matrices transforming Stokes parameters [4]. Furthermore, the structure of a general pure Mueller matrix, i.e. a 4×4 matrix transforming Stokes parameters that can be derived from a 2×2 matrix transforming electric field components, was investigated in a systematic way [5]. The results of all of these papers could be used not only for purely theoretical purposes, but also for detecting errors and inaccuracies in numerical and experimental results. This became increasingly important as it grew possible to study light scattering

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by nonspherical particles by advanced computational and experimental means. So, on the occasion of the first international workshop on light scattering by nonspherical particles held in Amsterdam in 1995, we decided to present a concise review of general existence tests for scattering matrices for arbitrary particles, orientations and scattering geometries [6]. Special attention was paid to simplicity, convenience and/or completeness. A more extended paper on the subject, entitled “Testing scattering matrices: a compendium of recipes,” was published in the special issue of *JQSRT* devoted to the workshop [7] and is reprinted in this issue unchanged. Since then we have reported coherent overviews of our work on basic relationships for matrices describing scattering by particles and also extended the work to multiple scattering matrices like the reflection and transmission matrices of plane-parallel media [8,9].

Several researchers have made good use of our paper reprinted in this issue to test their numerical or experimental results. The strongest tests can only be performed when all elements of the scattering matrix have been determined. In this way errors of various types in the electronic equipment as well as in the optical components and their alignment during the experimental determination of the angular distributions of scattering matrices of randomly oriented particles could be spotted and remedied.

The tests presented in [7] are valid for arbitrary directions of incident and scattered radiation. Scattering matrices for strictly forward and backward scattering can usually not be determined experimentally, but only numerically. For these special cases specific tests have been developed [10,11]. The tests for backward scattering by particles that are characterized by strong forward scattering appeared to be rather sensitive to inaccuracies in numerical computations. The special tests for strictly forward and backward scattering are also useful for extrapolating experimental data to very small and very large scattering angles [12].

For more recent work on interrelations of elements of matrices transforming Stokes or similar parameters as a result of scattering or other processes we refer to [13] and references therein. We believe, however, that our

paper [7] is still useful for many practical problems of scattering of light and other electromagnetic radiation by particles.

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