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Integral Equations and Operator Theory

Errata: Weighting Operator Patterns of Pritchard-Salamon Realizations

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In [1] the weighting pattern k_θ of a Pritchard-Salamon (PS) realization θ was introduced as the operator-valued function

$$k_\theta : \mathbb{R}^+ \to \mathcal{L}(U, Y), \qquad k_\theta(t)u = (\Lambda_\theta Bu)(t),$$

where Λ_θ is the so-called observability operator of θ. This definition is correct if at least one of the spaces U and Y is finite dimensional. However, as was pointed out to us by K. Mikkola (see also [3] for an example), it may happen that k_θ is not a function whose values are bounded operators from the input Hilbert space U into the output Hilbert space Y when these spaces are infinite dimensional. Therefore k_θ should have been introduced instead as the function from $\mathbb{R}^+ \times U$ into Y defined by

$$k_\theta(t, u) = (\Lambda_\theta Bu)(t), \qquad t \in \mathbb{R}^+ \text{ a.e.}, \tag{1}$$

where $k_\theta(\cdot, u) \in L_{2,loc}(\mathbb{R}^+; Y)$ for each $u \in U$.

The main theorem of [1] is the following result.

Theorem 1. *Let U and Y be complex Hilbert spaces, and let $k(\cdot) : \mathbb{R}^+ \to \mathcal{L}(U, Y)$. In order that $k(\cdot)$ is the weighting pattern of a PS-realization it is necessary and sufficient that for some $\mu \in \mathbb{R}$ the following hold:*

$$e^{\mu\cdot} k(\cdot)u \in L_2(\mathbb{R}^+, Y) \qquad (u \in U), \qquad e^{\mu\cdot} k(\cdot)^* y \in L_2(\mathbb{R}^+, U) \qquad (y \in Y),$$

where the asterisk denotes the adjoint.

Theorem 1 is correct as stated. However, from Mikkola's example referred to above it follows that for U and Y infinite dimensional Theorem 1 does not provide a complete characterization of weighting patterns of arbitrary PS-realizations as claimed in [1]. To get such a characterization, first recall from [1] that the adjoint

θ^* of a PS-realization θ is well defined and is again a PS-realization. Moreover, one can show that for each $u \in U$ and $y \in Y$ we have

$$\langle k_\theta(\cdot, u), y\rangle = \langle u, k_{\theta^*}(\cdot, y)\rangle \qquad \text{a.e. on } \mathbb{R}^+.$$

Hence in order to derive necessary and sufficient conditions for k to be the weighting pattern of an arbitrary PS-realization we have to replace the operator functions $k(\cdot)$ and $k(\cdot)^*$ appearing in Theorem 1 by vector valued functions $k : \mathbb{R}^+ \times U \to Y$ and $k_* : \mathbb{R}^+ \times Y \to U$ related by

$$\langle k(\cdot, u), y\rangle = \langle u, k_*(\cdot, y)\rangle \qquad \text{a.e. on } \mathbb{R}^+,$$

where $u \in U$ and $y \in Y$. The following theorem characterizes weighting patterns of PS-realizations.

Theorem 2. *Let U and Y be complex Hilbert spaces, and let $k : \mathbb{R}^+ \times U \to Y$. In order that k is the weighting pattern of a PS-realization it is necessary and sufficient that there exist a function $k_* : \mathbb{R}^+ \times Y \to U$ and $\mu \in \mathbb{R}$ such that for each $u \in U$ and $y \in Y$ the following hold:*

$$\langle k(\cdot, u), y\rangle = \langle u, k_*(\cdot, y)\rangle \qquad a.e. \;\; on\; \mathbb{R}^+, \tag{2a}$$

$$e^{\mu\cdot} k(\cdot, u) \in L_2(\mathbb{R}^+, Y), \qquad e^{\mu\cdot} k_*(\cdot, y) \in L_2(\mathbb{R}^+, U). \tag{2b}$$

The proof of Theorem 2 follows the same line of reasoning as that of Theorem 1.1 in [1] provided that at appropriate places one replaces the operator functions $k(\cdot)$ and $k(\cdot)^*$ by the vector functions $k(\cdot,\cdot)$ and $k_*(\cdot,\cdot)$. Similar modifications are to be made in part (4) of Lemma 1.3, Propositions 3.1 and 4.1, Theorem 4.2, and Corollary 4.3 of [1].

The definition of the weighting pattern of an extended Pritchard-Salamon realization given in [2] should be modified in the same way. The adjoint in Theorem 4.4 of [2] is to be interpreted as in (2a). None of the results of [2] is affected by this change of definition.

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