

Reply to referee report on:
PARAMETER ESTIMATION OF
MONOMIAL-EXPONENTIAL SUMS
by L. Fermo, C. van der Mee, and S. Seatzu

First of all, we would like to express our appreciation for the effort invested in writing the referee report. Nevertheless, we have a different assessment of some of the issues raised by the referee.

The major differences between our paper and the five papers referred to by the referee are the following:

1. We choose to recover the parameters of the monomial-exponential sum by using more data than parameters. Thus, using the matrix pencil method which involves Hankel matrix systems, we arrive at rectangular rather than quadratic matrix systems. As a result, we need to use the GSVD method.¹ In the five papers cited the number of data is always the same as the number of parameters. In the application that has inspired our research (computing the spectral data of the Zakharov-Shabat system and solving the Cauchy problem of the nonlinear Schrödinger equation), it is pretty easy to generate more data than there are parameters, but using more data than parameters leads to more accurate results.
2. We consider the case of multiple zeros (or zeros differing by a small amount), where the corresponding Vandermonde matrix is either singular or illconditioned. In the application that has inspired our research, multiple zeros or single zeros close together are quite normal.

¹In the new version of our paper, we propose to explain this issue in more detail.

3. When rendering proofs of theoretical results, we rely on the theory of linear higher order difference equations, where the so-called Casorati matrix plays the role of the Wronskian matrix. One advantage of this approach, not formulated in the paper, is that it leads to an immediate generalization to systems of monomial-exponential sums, something of great interest to integrable systems (such as the Manakov system generalizing the Zakharov-Shabat system presented in the paper).

Thus in our opinion we have presented results going far beyond the contents of the five papers indicated by the referee.

Let us consider our major example: nonlinear PDE's of integrable type. The referee may consider them "very interesting," but this application actually generates numerical problems not considered in previous papers and considered in our paper: more data than parameters (to increase accuracy), multiple zeros or close together simple zeros. We were baffled by the fact that none of the five predecessor papers ever pointed out the relationship with the theory of higher order linear difference equations, a fact that has led to more complicated proofs of certain theoretical results in their papers. It is only natural that we are asking for a reassessment of the referee report.