Cornelis VAN DER MEE, Spring 2008, Math 3330, Exam 4

Name:................................**Grade**:..........**Rank**:...........

To receive full credit, show all of your work. Neither calculators nor computers are allowed.

ex.1	ex.2	ex.3	ex.4	ex.5	ex.6	ex.7	S1	S2	S3	S4

1. Compute the eigenvalues and corresponding eigenvectors of the matrix
$$A = \begin{pmatrix} 6 & 1 \\ 3 & 8 \end{pmatrix}.$$
Use this information to diagonalize the matrix A if possible. Otherwise indicate why diagonalization is not possible.

2. Find a 2×2 matrix A such that $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ are eigenvectors of A, with eigenvalues -2 and 1, respectively.

3. Consider the discrete dynamical system
$$x(n+1) = Ax(n), \qquad n = 0, 1, 2, 3, \ldots,$$
where
$$A = \begin{pmatrix} 3 & -3 \\ 1 & -1 \end{pmatrix}, \qquad x(0) = \begin{pmatrix} 2 \\ 0 \end{pmatrix}.$$
   a. Write $x(0)$ as a linear combination of eigenvectors of A.
   b. Compute $x(n)$ for $n = 1, 2, 3, \ldots$.

4. Find all eigenvalues (real and complex) of the matrix
$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & 3 \end{pmatrix}.$$
Explain why or why not the matrix A is diagonalizable. Solution: The

5. Compute the eigenvalues and corresponding eigenvectors of the matrix

$$A = \begin{pmatrix} 1 & 5 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{pmatrix}.$$

   Use this information to diagonalize the matrix A if possible. Otherwise indicate why diagonalization is not possible.

6. Consider the matrix

$$A = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

   a. Compute the eigenvalues (real and complex) of the matrix A.
   b. Compute the **algebraic** multiplicities of these eigenvalues.
   c. Explain why your result is in full agreement with the values of $\mathrm{Tr}(A)$ and $\det(A)$.