Cornelis VAN DER MEE, Spring 2008, Math 3330, Exam 2

Name:..............................**Grade**:..........**Rank**:...........

To receive full credit, show all of your work. Neither calculators nor computers are allowed.

ex.1	ex.2	ex.3	ex.4	ex.5	ex.6	ex.7	S2

1. Consider the following 5×7 matrix:

$$A = \begin{pmatrix} 0 & 1 & 2 & 0 & 0 & 1 & 0 \\ 1 & 3 & 6 & 0 & 4 & 2 & 0 \\ 0 & 0 & 0 & 1 & 0 & 2 & 3 \\ 0 & 0 & 0 & 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

   a. Find a basis of the image of A and show that it really is a basis.

   b. Find a basis of the kernel of A and show that it really is a basis.

   c. Illustrate the rank-nullity theorem using the matrix A.

2. Consider the following 4×4 matrix:

$$A = \begin{pmatrix} 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 1 & 0 & 0 & -2 \end{pmatrix}.$$

   a. Find a basis of the image of A and show that it really is a basis.

   b. Find a basis of the kernel of A and show that it really is a basis.

   c. Illustrate the rank-nullity theorem using the matrix A.

3. Consider the following four vectors:

$$\vec{v}_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \\ 0 \end{pmatrix}, \quad \vec{v}_2 = \begin{pmatrix} 1 \\ 0 \\ -2 \\ -4 \end{pmatrix}, \quad \vec{v}_3 = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}, \quad \vec{v}_4 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

a. Argue why or why not $S = \{\vec{\boldsymbol{v}}_1, \vec{\boldsymbol{v}}_2, \vec{\boldsymbol{v}}_3, \vec{\boldsymbol{v}}_4\}$ is a linearly independent set of vectors.

b. If S is not a linearly independent set of vectors, remove as many vectors as necessary to find a basis of its linear span and write the remaining vectors in S as a linear combination of the basis vectors.

4. Find the rank and nullity of the orthogonal projection onto the hyperplane $x_1 - x_2 + x_3 - x_4 = 0$ in $\mathbb{R}^4$. Argue why your result is correct.

5. Compute the matrix of the linear transformation

$$T(\vec{\boldsymbol{x}}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \vec{\boldsymbol{x}}, \qquad \text{where } \vec{\boldsymbol{x}} \in \mathbb{R}^3,$$

with respect to the basis

$$\vec{\boldsymbol{v}}_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \qquad \vec{\boldsymbol{v}}_2 = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}, \qquad \vec{\boldsymbol{v}}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

6. Argue why or why not the set of polynomials

$$1 + x^2, \qquad x - x^3, \qquad 1 - x^2, \qquad x + x^3, \qquad x^4,$$

is a basis of the vector space of polynomials of degree ≤ 4.

7. Find a basis of the vector space of all 2×2 matrices S for which

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} S = S \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$