Cornelis VAN DER MEE, Spring 2008, Math 3330, Exam 1

Name:. . . . . **Grade**:. . . . . **Rank**:. . . . .
To receive full credit, show all of your work. Neither calculators nor computers are allowed.

1. Bring the following matrix to reduced row echelon form:

$$A = \begin{pmatrix} 1 & 3 & 0 & 5 & 3 \\ 0 & 0 & 1 & 4 & -1 \\ 0 & 0 & 1 & 3 & 2 \\ 0 & 0 & 0 & 0 & 5 \end{pmatrix},$$

   and determine its rank.

2. If the augmented matrix for a nonhomogeneous system of linear equations has been reduced by row operations to the matrix

$$\left(\begin{array}{ccccc:c} 1 & 7 & 0 & 0 & -3 & 11 \\ 0 & 0 & 2 & 0 & 8 & -10 \\ 0 & 0 & 0 & 1 & -6 & 7 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right),$$

   what is the solution to this linear system?

3. Find **all** solutions of the linear system $A\vec{\boldsymbol{x}} = \mathbf{0}$, where

$$A = \begin{pmatrix} 2 & 3 & 6 \\ 0 & 1 & -1 \end{pmatrix}.$$

4. Evaluate the inverse of the matrix

$$A = \begin{pmatrix} 11 & 7 \\ -4 & -3 \end{pmatrix}.$$

5. Find the matrix A such that

$$A\begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ 2 \end{pmatrix}, \qquad A\begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}, \qquad A\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -4 \\ 5 \end{pmatrix}.$$

6. Determine the matrix of the projection of any point $\vec{\boldsymbol{x}} \in \mathbb{R}^3$ onto the line through the origin and the point $(3, 4, 0)$.

7. Determine the matrix of the counterclockwise rotation in $\mathbb{R}^2$ through the angle $\theta = 45°$.

8. Find the inverse of the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 6 \\ 0 & 4 & 5 \end{pmatrix}.$$

9. Compute A^3, where $A = \begin{pmatrix} 1 & a \\ 0 & 3 \end{pmatrix}$ and a is a parameter.

10. Compute the matrix product ABC, where

$$A = \begin{pmatrix} 5 & 7 & 0 \\ 2 & -3 & 1 \\ 0 & 0 & 2 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & -5 & 3 \\ 0 & 3 & 2 \\ 0 & -2 & 3 \end{pmatrix}, \qquad C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix}.$$