Cornelis VAN DER MEE, Spring 2008, Math 3330, Sample Final Exam

Name:...............................**Grade**:..........**Rank**:...........
To receive full credit, show all of your work. Neither calculators nor computers are allowed.

1. Bring the following matrix to reduced row echelon form:

$$A = \begin{pmatrix} 0 & 0 & 1 & 3 & -7 \\ 1 & -5 & 0 & 2 & 4 \\ 0 & 0 & 2 & 6 & -2 \\ 0 & 0 & 0 & 0 & 9 \end{pmatrix},$$

   and determine its rank and nullity.

2. Find **all** solutions of the linear system $A\vec{\boldsymbol{x}} = \mathbf{0}$, where

$$A = \begin{pmatrix} 3 & 6 & 7 \\ 0 & 2 & -1 \end{pmatrix}.$$

3. Consider the following 4×4 matrix:

$$A = \begin{pmatrix} 0 & 1 & 1 & 3 \\ 1 & 0 & 0 & -2 \\ 0 & 0 & 2 & 6 \\ 0 & 2 & 0 & 0 \end{pmatrix}.$$

   a. Find a basis of the image of A and show that it really is a basis.
   b. Find a basis of the kernel of A and show that it really is a basis.

4. Argue why or why not the set of polynomials

$$1, \qquad x, \qquad 2x^2 - 1, \qquad 4x^3 - 3x, \qquad 8x^4 - 8x^2 + 1,$$

   is a basis of the vector space of polynomials of degree ≤ 4.

5. Find the matrix of the orthogonal projection of $\mathbb{R}^4$ onto the hyperplane

$$x_1 - 2x_2 + 3x_3 - 4x_4 = 0.$$

6. Apply the Gram-Schmidt process to the given basis vectors of

$$V = \text{span}\left[\begin{pmatrix}3\\0\\4\\0\end{pmatrix}, \begin{pmatrix}0\\0\\5\\0\end{pmatrix}, \begin{pmatrix}4\\0\\0\\3\end{pmatrix}\right]$$

to obtain an orthonormal basis of V.

7. Find a least-squares solution to the system

$$\begin{pmatrix}1 & 0\\2 & 1\\0 & 3\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix} = \begin{pmatrix}2\\0\\1\end{pmatrix}.$$

8. Find the factors Q and R in the QR factorization of the matrix

$$M = \begin{pmatrix}1 & 0\\2 & 4\\1 & -2\end{pmatrix}$$

by using the Gram-Schmidt process.

9. Find the determinant of the 3×3 matrix

$$A = \begin{pmatrix}1 & 2 & -3\\2 & 0 & 5\\2 & 6 & 4\end{pmatrix}.$$

10. Compute the eigenvalues and corresponding eigenvectors of the matrix

$$A = \begin{pmatrix}5 & 3\\1 & 7\end{pmatrix}.$$

Use this information to diagonalize the matrix A if possible. Otherwise indicate why diagonalization is not possible.

11. Find all eigenvalues (real and complex) of the matrix

$$A = \begin{pmatrix}0 & 0 & 1\\1 & 0 & 0\\-9 & -9 & -1\end{pmatrix}.$$

Why or why not is it possible to diagonalize the matrix A?

12. Find the solution of the discrete dynamical system

$$x(n+1) = Ax(n), \qquad n = 0, 1, 2, 3, \dots,$$

where

$$A = \begin{pmatrix} 3 & 4 \\ 4 & -3 \end{pmatrix}, \qquad x(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$