Cornelis VAN DER MEE, Spring 2008, Math 3330, Sample Exam 3

Name:.................................**Grade**:..........**Rank**:...........

To receive full credit, show all of your work. Neither calculators nor computers are allowed.

1. Consider the two vectors

$$\vec{\boldsymbol{u}} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \qquad \vec{\boldsymbol{v}} = \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix}.$$

   a. Compute the cosine of the angle between $\vec{\boldsymbol{u}}$ and $\vec{\boldsymbol{v}}$.

   b. Compute the distance between $\vec{\boldsymbol{u}}$ and $\vec{\boldsymbol{v}}$.

   c. Does there exist an orthogonal 3×3 matrix A such that $A\vec{\boldsymbol{u}} = \vec{\boldsymbol{v}}$? If it exists, construct one. If it does not exist, explain why not.

2. Find an orthonormal basis for

$$V = \text{span}\left[\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 3 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}\right]$$

   and use this information to write down the orthogonal projection of $\mathbb{R}^4$ onto V.

3. Find a least-squares solution to the system

$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

3.* (**TO BE REPLACED**) Find the factors Q and R in the QR factorization of the matrix

$$M = \begin{pmatrix} 1 & 0 \\ 2 & 4 \\ 1 & -2 \end{pmatrix}$$

   by using the Gram-Schmidt process.

4. Find the determinant of the 3×3 matrix

$$A = \begin{pmatrix} 1 & 1 & -5 \\ 0 & 2 & 4 \\ 3 & 6 & 9 \end{pmatrix}.$$

Describe the parallelepiped whose volume is given by this determinant.

5. Find the determinants of the 4×4 matrices

$$A = \begin{pmatrix} 0 & 0 & 0 & 3 \\ 0 & 0 & 1 & 3 \\ 0 & 2 & 5 & -1 \\ 4 & 2 & 8 & -2 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix}.$$

6. Let A be a 7×7 matrix with $\det(A) = -3$.

   a. Compute $\det(-2A)$.
   b. Compute $\det(AA^T)$.
   c. Compute $\det(A^T A^{-1})$.
   d. Compute the determinant of the matrix obtained from A by first interchanging the last two columns and then interchanging the first two rows.