

Cornelis VAN DER MEE, Spring 2008, Math 3330, Sample Exam 1

Name: Grade: Rank:

To receive full credit, show all of your work. Neither calculators nor computers are allowed.

1. Bring the following matrix to reduced row echelon form:

$$A = \begin{pmatrix} 0 & 0 & 0 & 1 & 2 & -6 \\ 1 & 2 & 0 & 0 & 1 & -1 \\ 1 & 2 & 2 & 0 & -1 & 1 \end{pmatrix},$$

and determine its rank.

2. If the augmented matrix for a nonhomogeneous system of linear equations has been reduced by row operations to the matrix

$$\left(\begin{array}{cccc|c} 1 & 3 & 0 & 0 & -5 & 9 \\ 0 & 0 & 2 & 0 & 4 & -6 \\ 0 & 0 & 0 & 3 & -6 & 9 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right),$$

what is the solution to this linear system?

3. Find all solutions of the linear system $A\vec{x} = \mathbf{0}$, where

$$A = \begin{pmatrix} 3 & 4 & 7 \\ 0 & -1 & 2 \end{pmatrix}.$$

4. Evaluate the inverse of the matrix

$$A = \begin{pmatrix} 9 & 8 \\ -5 & 7 \end{pmatrix}.$$

5. Find the matrix A such that

$$A \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -6 \\ 2 \end{pmatrix}, \quad A \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} -6 \\ -4 \end{pmatrix}, \quad A \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 8 \\ 3 \end{pmatrix}.$$

6. Determine the matrix of the projection of any point $\vec{x} \in \mathbb{R}^2$ onto the line through the origin and the point $(-5, 12)$.
7. Determine the matrix of the counterclockwise rotation in \mathbb{R}^2 through the angle $\theta = 60^\circ$.
8. Find the inverse of the matrix

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 4 & 0 \\ 2 & 7 & 1 \end{pmatrix}.$$

9. Compute A^4 , where $A = \begin{pmatrix} 1 & 0 \\ a & 3 \end{pmatrix}$ and a is a parameter.
10. Compute the matrix product ABC , where

$$A = \begin{pmatrix} 6 & 3 & 0 \\ 4 & -1 & 2 \\ 0 & 0 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & -4 & 5 \\ 0 & 2 & 1 \\ 0 & -6 & 2 \end{pmatrix}.$$